Fig.3 shows the relation $\delta(t)$ for fixed values of μ, for the cases of natural and forced aging (the solid and dashed lines, respectively). The function $\delta(t)$ increases with time t and tends to a limiting value, which is larger, the larger the parameter μ.

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STEADY STATE BOUNDARY FLOWS IN THE LIGHT OF THE GENERALIZED KARMAN THEORY*

V.V. NOVOZHILOV

Results are given, based on the generalization in /1/ of the Karman theory of turbulence, obtained within the last ten years. The advantages and disadvantages of the model of turbulent flows used are analyzed and comparisons are made with other models.

1. Blasius's empirical formula of (1911) represents the first significant success in the applied theory of turbulence

$$\lambda \, \mathrm{Re}^{1/4} = 0{,}316 \qquad (1.1)$$

The formula expresses the dependence of the coefficient of resistance on the Reynolds number in steady state flow in a straight pipe of circular cross-section. However, the relationship had no connection with the Reynolds equation and was therefore considered to represent an achievement in hydraulics rather than hydrodynamics. In the 1920-s Prandtl proposed, while developing the Reynolds' and Bussinesq's ideas, the phenomenological theory of turbulent steady-state flows, i.e. the mixing-length theory.

In fact, the problem was that of constructing a model of a non-linearly viscous fluid the laminar flow of which would be identical (in velocity profiles and stress distribution) with the averaged turbulent flow (with analogous boundary conditions). Prandtl's idea was complemented by Karman who put forward the idea of the selfsimilarity of steady-state turbulent flows. As a result a solution was obtained for the averaged turbulent flow in a straight pipe of circular cross-section, as well as results for the velocity profiles, and the relation $\lambda = f(\mathrm{Re})$, which agreed well with experimental data. The latter relation was practically

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identical with Blasius' formula (1.1) in the range ($2\cdot10^3 < \mathrm{Re} < 2\cdot10^5$) of moderate values of the Reynolds number.

While trying to explain this fact, Karman found that the power formulas

$$\lambda \mathrm{Re}^{1-n} = A_n = \mathrm{const} \tag{1.2}$$

could be obtained provided that we assumed that the averaged flow velocity profile $u(y)$ is given by the expression

$$\frac{u}{U_m} = \left(\frac{y}{r_0}\right)^k, \quad k = \frac{1-n}{1+n} \tag{1.3}$$

where r_0 is the pipe radius, U_m is the maximum averaged velocity, and y is the distance from the pipe wall. It was also found that within the range of the values of Re in which Blasius' formula (obtained from (1.2) at $n = 3/4$, $A_n = 0.316$), holds, the velocity profile (1.3) is fairly close to the universal logarithmic profile following from the mixing-length theory.

Usually the arguments used by Karman in deriving the power profile (1.3) are not quite regarded as being the derivation of this profile from (1.2). However, in fact Karman has only shown that (1.3) does not contradict (1.2). But this applied to any other velocity profile asymptotically close to (1.3) as $y \to 0$. It is essential that a class of flows for which a relation of the type (1.2) holds (over a certain, fairly large range of values of the Reynolds number), is much greater than the class of flows for which the averaged velocity is approximated by the power law (1.3).

Formulas (1.2) can be successfully used to interpret experiments for numerous steady-state turbulent flows. These include, in particular, the selfsimilar plane turbulent boundary layers with positive and negative pressure gradients /1/, flows between two coaxial rotating cylinders /2,3/, steady flow in a curved channel /4/ and certain magnetohydrodynamic flows /5/. For all these flows the velocity profiles cannot be represented in the form of a power relation of the type (1.3).

In this connection the following problem naturally arises: to find a closure of the Reynolds equations, i.e. a relation connecting the Reynolds stresses with the mean velocity derivatives, which will lead to relations of the type (1.2) connecting the resistance coefficient with the Reynolds number. Such a result would be equivalent to deriving Blasius' formula from the Reynolds equations, i.e. to converting it from an empirical to a phenomenological type relationship.

2. This method of formulating the problem places it in the initial period of theoretical investigation of the turbulence. Its solution however was delayed for almost 50 years. It is given by the following formula for the turbulent viscosity ν_τ:

$$\frac{\nu_\tau}{\nu} = \varkappa_n T^n, \quad T = \left|\frac{\partial u}{\partial y}\right|^3 : \nu \left|\frac{\partial^2 u}{\partial y^2}\right|^2 \tag{2.1}$$

where ν is the molecular viscosity, and n and $\varkappa_n$ are dimensionless physical constants.

Expression (2.1) includes, as two special cases, the formulas for a linearly viscous fluid ($n = 0, \varkappa_0 = 1$) and for the Karman turbulent viscosity ($n = 1, \varkappa_1 = 0.16$). In these two limiting cases Eq. (2.1) does not, of course, contribute anything new. The intermediate values $0 < n < 1$ however yield interesting and non-trivial results. In this case it is found to be possible to impose on the solution not only the Karman condition $\partial u/\partial y \to \infty$, but also the condition of adhesion $u = 0$, obviating in this manner the need for a viscous sublayer. In the case of one-dimensional and selfsimilar problems the use of (2.1) leads to an expression of the form (2.1) connecting the coefficients of resistance with the Reynolds numbers. In this case, however, the velocity profiles show no power type dependence and cannot be described by formulas of the type (1.3) except in the single special case of a flow at a flat wall under the action of a constant tangential force. In all the remaining cases the velocity profiles are more complicated and approach (1.3) asymptotically only in the immediate vicinity of the wall.

3. Let us note some basic properties of the theory following from formula (2.1). At first sight it can be used in cases when the velocity profiles of the averaged flow have points of inflection (e.g. in Couette flow). Indeed, at the point of inflection $\partial^2 u/\partial y^2 = 0$, therefore according to (2.1) the turbulent viscosity at this point ought to tend to infinity, which clearly contradicts reality. The contradiction can be overcome if we assume that the profile curvature at the point of inflection does not pass through zero continuously, but suffers a finite discontinuity $\partial^2 u/\partial y^2 = \pm a$ there. Then, according to (2.1) the turbulent viscosity at the point of inflection will not only be finite, but also continuous as required.

Solutions of numerous specific problems /3/ show that the velocity profiles and coefficients of resistance obtained under these assumptions agree well with experimental data without the need to alter the experimental constants n and $\varkappa_n$ (determined earlier, once and for always from experimental investigations of flows in pipes).

The latter support the assumption adopted, although some workers regard it as an artificial case without any physical basis. However, if we take into account the fact that any steady state flow with a point of inflection in its velocity profile is formed as a result of the merging of two boundary layers (or streams), then the appearance and retention of a weak singularity in the form of a discontinuity in the curvature of its velocity profile, along their lines of contact, is physically justified, and this is confirmed by numerous comparisons of the computational and experimental data.

Note that in solving within the framework of the mixing-length theory, turbulent flows with inflection in the velocity profiles, it is usually necessary to make special assumptions of one sort or another regarding the length of the mixing path (see e.g. /6,7/) and to introduce new empirical constants. The undoubted advantage of this method of solving such flows lies in its uniformity, i.e. in preserving for all problems the same initial formulas and the same empirical constants derived from the Nikuradze experiments /1/.

The points at which the curve of the tangential stresses changes its sign are, according to the theory, angular points for the velocity profiles. Profiles containing angular points are not new in the theory of turbulence. For example, the mixing-length theory in its classical form, when applied to flow in a straight pipe, leads to velocity profiles which are somewhat peaked on the pipe axis. It should be stressed that a tendency to such peaking can be observed on all experimental profiles obtained for the flows in pipes by Nikuradze and others. The peaking can be observed on the graph itself, and was called, in the hydraulics at the beginning of this century, a "D'Arcy cap". We recall that the so-called first Prandtl Theory for streams and wakes leads to pointed profiles. All this is also present in the generalized Karman theory (2.1). In particular, the theory of selfsimilar plane boundary layers following from it leads to the appearance of corner points at the free boundary of the layer, i.e. the averaged velocity does not couple smoothly with the velocity of the potential flow /1/. Perusal of the voluminous atlas of experimental data on velocity profiles in boundary layers given in the second volume of the proceedings of the Stanford conference /8/ confirms this.

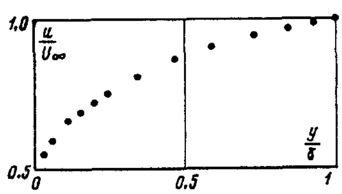

Fig.1

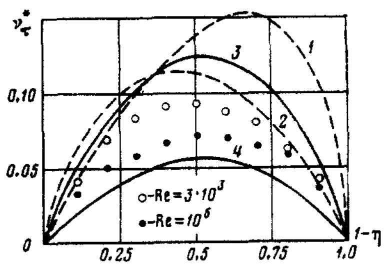

Fig.2

Now is the time to mention that in discussing the paper by A.M. Kolmogorov /9/, L.D. Landau showed that "when dealing with the equations of turbulent motion, we must always take into account the fact that the presence of vorticity in the turbulent stream is restricted by the finite size of the space; qualitatively correct equations must lead to such a distribution of the vortices". The Landau condition means basically that a sharp boundary separating the regions of potential and turbulent flows is necessary. This is satisfied, in particular, by Prandtl's first theory of streams and wakes, but not by his second theory. The generalized Karman theory (2.1) always leads to a sharp separation of the regions of potential and turbulent flows, thus satisfying the Landau postulate.

In fact that the proposed theory leads to the appearance of corner points on the velocity profiles at points where the tangential stresses change sign, conforms with the theory. As we know, the fact that the tangential Reynolds stresses become zero does not imply that there is no variation in the velocity. Turbulent motion occurs at such points with sufficient intensity and has an alernating character. This means that the points cannot always be detected experimentally and the pattern often appears diffuse /10/. However, a large volume of experimental data /8/ exists which indicate that the corner points on the velocity profiles of the turbulent flows are an objective fact. This is clearly seen in Fig.1, which shows the results of experiments id 2100 (boundary layer flow; U_∞ is the velocity at the outer boundary of the boundary layer, and δ denotes its thickness).

4. Formula (2.1) for turbulent viscosity contains the molecular viscosity ν in explicit form. This sometimes gives rise to objections, since the formulas for ν_τ obtained earlier did not usually contain the molecular viscosity, arguing that trubulent mixing away from the wall (without a viscous sublayer) must obey the laws of flow of a perfect fluid. Nevertheless, the experimental determination of ν_τ with help of the formula

$$-\langle u'v'\rangle = \nu_\tau \partial u/\partial y \tag{4.1}$$

where $\langle u'v'\rangle$ and $\partial u/\partial y$ are measured, shows that ν_τ depends essentially on the Reynolds number and hence on ν.

In the classical theory of turbulence the contradiction is overcome by introducing the concept of a viscous sublayer the thickness of which is given by the empirical formula

$$v_* h/\nu = 11 \tag{4.2}$$

where v_* is the dynamic velocity. The condition that the velocity is continuous on the boundary between the viscous sublayer and the region of turbulent flow, implies that the turbulent viscosity depends, after all, in an indirect manner, on the molecular viscosity.

In the generalized Karman theory $(n \neq 1)$ it becomes possible to satisfy the condition of adhesion of fluid at the boundary without resorting to the concept of a viscous sublayer. This is equivalent to neglecting the thickness of this sublayer. But then we must include ν in the formula for the turbulent viscosity, which is indeed done in (2.1), We shall use the classical problem of turbulent flow in pipes to show that such an approach is admissible and yields correct results, by considering and comparing various formulas.

Using the mixing-length theory and putting $l = 0.4 r_0 (1-\eta)$, where $\eta = r/r_0$, r_0 is the pipe radius, we can obtain

$$\nu_\tau^* = \frac{\nu_\tau}{v_* r_0} = 0.4\,(1-\eta)\,\eta^{1/2} \tag{4.3}$$

while (2.1) with $n = 1, \varkappa_n = 0.16$ (which corresponds to the Karman theory), yields the expression

$$\nu_\tau^* = 0.8\,(1-\eta^{1/2})\,\eta \tag{4.4}$$

The same formula (2.1) but with $n = 3/4, \varkappa_n = 0.53$ ([1]), gives

$$\nu_\tau^* = \frac{2{,}66}{\mathrm{Re}^{1/8}}\,(1-\eta^{1/8})^{8/7}\,\eta \tag{4.5}$$

Nikuradze determined the function $\nu_\tau^*(\eta)$ independently of any hypotheses regarding the value of the mixing length, using the formula

$$\nu_\tau^* = -v_*\eta : \frac{du}{d\eta}, \quad v_*^2 = \frac{qr_0}{2\rho} \tag{4.6}$$

where the pressure gradient q and du/dy were determined experimentally.

Fig.2 shows curves of $\nu_\tau^* = f(1-\eta)$ constructed using (4.3) (curve *1*) and (4.4) (curve *2*); curves *3* and *4* correspond to the Reynolds number $\mathrm{Re} = 3\cdot 10^3$ and $\mathrm{Re} = 10^6$ respectively. We see that the classical formulas (4.3) and (4.4) yield substantially higher values of ν_τ, and (4.4), following from the Karman theory, gives slightly better results. But even the latter exceeds the experimental values by 50% at $\mathrm{Re} = 3\cdot 10^3$ and by 27% at $\mathrm{Re} = 10^6$. The results obtained for ν_τ^* from the proposed formula (3.3) are higher than the experimental data by 38% at $\mathrm{Re} = 3\cdot 10^3$ and lower by 13% at $\mathrm{Re} = 10^6$. It should be noted however that the values $n = 3/4$, $\varkappa_n = 0.53$ of the constants are intended only to be used for calculations falling within the "Blasius" range of Reynolds numbers, and at $\mathrm{Re} = 10^5$ Eq.(4.5) yields a curve for ν_τ^*, which is practically identical with the experimental curve and with that given by the formula

$$\nu_\tau^* = (0{,}14 - 0.08\eta^2 - 0.06\eta^4)\,\eta^{1/2} \tag{4.7}$$

which is obtained when the empirical Prandtl formula based on the Nikuradze experiments is used for the mixing length. The dependence of the constants n and $\varkappa_n$ used in the theory (2,1) on the Reynolds numbers, has already been discussed and pairs of their values for various ranges of Re are given in /1/.

5. The above comparison is fairly typical and shows that molecular viscosity ν_τ can be brought in indirectly and yields results no worse than those obtained by introducing ν into ν_τ through the boundary condition, as has been done up to now. At the same time, the scope for solving turbulent flows is increased considerably. The proposed theory was successfully applied in /1–5/ to various problems mentioned in Sect.1.

The results obtained in 1978 for Couette flow with (and without) a pressure gradient are of interest, since the problem can be solved within the framework of the proposed theory in quadratures. Its solutions obtained within the framework of traditional representations /6, 7,11/ were derived by choosing specially the formula for the mixing path and for the physical constants appearing in it, i.e. the formula was tailored to the problem in question. Meanwhile, using the generalized Karman theory (2.1) the problem can be solved in closed form without altering the initial formula and the constants appearing in it.

A new solution to the Hartman problem /5/ also did not require any changes in (2.1), nor in the values of the constants. Its transpired that a transverse magnetic field affects the flow only in terms of its averaged characteristics. The laws governing turbulent mixing remain the same as for non-electrically conducting liquids. Until now the theory of turbulent magnetohydrodynamic flows was regarded as an independent branch of the theory of turbulence, requiring a revision of the formulas for turbulent viscosity (see e.g. /12/).

Special attention must be given to the results obtained for curvilinear flows. Since the time of Prandtl's work /13/ it has been known that the computations must take into account the substantial influence of centrifugal forces on turbulent mixing. This problem however has not been tackled theoretically. The following generalization of the theory (2.1) to embrace curvilinear flows described in a polar coordinate system, was proposed in /3/:

$$\frac{\nu_\tau}{\nu} = \varkappa_n f(\mathrm{Ri})\, T^n \tag{5.1}$$

$$T = \left| r\frac{d\omega}{dr}\right|^3 : \nu\left|\frac{d}{dr}\left(r\frac{d\omega}{dr}\right)\right|^2, \quad \mathrm{Ri} = \frac{2\theta(1+2\theta)}{(1+\theta^2)}, \quad \theta = \frac{\omega}{r d\omega/dr}$$

Here $\omega(r)$ is the angular velocity of the liquid, $f(\mathrm{Ri})$ is a correction coefficient allowing for the effect of centrifugal forces on the turbulent mixing, and Ri is the Richardson gradient number known in the theory of statified turbulent flows, and used in the form generalized to curvilinear Bradshaw flows /14/. The analogy between statified and curvilinear turbulent flows was noted in /15/.

Analyzing the large amount of experimental data on turbulent flows between rotating cylinders, we obtained in /3/ the expression

$$f(\mathrm{Ri}) = 1 - 1{,}1\mathrm{Ri}^{3/10} \tag{5.2}$$

valid for the Blasius range of Re for which $n = 3/4$, $\varkappa_n = 0.53$. The expression for the correction factor $f(\mathrm{Ri})$ was then successfully used to compute the flow in a curved channel /4/. Formulas were obtained for the coefficient of resistance and for the coordinate of the maximum of the velocity profile, which were in good agreement with experimental data. Formulas (5.1) and (5.2) open up the prospects of computing plane turbulent boundary layers on curvilinear boundaries, and increase the accuracy of the determination of their points of separation.

We note that in computing curvilinear turbulent flows the number of empirical constants is increased by two. Thus we have, in addition to $n = 3/4$, $\varkappa_n = 0.53$, another two constants in (5.2): $\beta = 1.1$, $\alpha = 3/10$. This means that the theory has reached its critical level of four empirical constants, beyond which every theory transforms, according to Landau, into a process of adjusting the formulas to fit the experimental data.

6. Formula (2.1) was also used to compute plane boundary layers with positive and negative pressure gradients /1/. The results obtained were checked against all 32 series of experiments with boundary layers described in /8/. The outcome of this exercise is described in detail in /1/, so we shall just mention here the most important results of these tests.

The best agreement with experiment (with regard to both velocity profiles and coefficients of resistance) was obtained for the following values of the physical constants $n = 2/3$, $\varkappa_n = 0.56$ and not for $n = 3/4$, $\varkappa_n = 0.53$, as might have been expected bearing in mind that it was the latter set of the values of the constants that gave best results in the remaining special cases.

This can obviously be explained not only by the fact that the boundary layers studied in /1/ were characterized by the relatively low Reynolds numbers, but also by the fact that experiments involving boundary layers with positive pressure gradients are usually prone to error because of the difficulty of maintaining the plane character of the flow. This is particularly true for layers which have become detached. The best results were obtained when computing the equilibrium boundary layers for which the theory gave excellent agreement with experiment, at the same time confirming the experimentally found non-uniqueness of the solution of the problem. Using these results, we have succeeded in generalizing the Buri equation by converting it to a Riccati-type equation with the coefficient determined not empirically, but from the theory (2.1).

7. All the problems listed here are those of the theory of boundary layer turbulence. The use of formula (2.1) to solve free turbulence problems is not inadmissible, but a problem arises here, still not fully solved, namely, what condition at the free boundary should replace the Karman condition $\partial u/\partial y \to \infty$, which holds only on the walls. In this case an additional condition must be formulated (despite specifying the velocity at the free boundary), since (2.1) contains not only the first, but also the second derivative of the velocity, and

this increases the order of the differential equation of flow by one (as compared with the version of the mixing-length theory in Prandtl form). At first glance it appears that the relation $\partial u/\partial y = 0$, which presupposes a smooth variation in the velocity at the boundary between the potential and turbulent flow can be used as the additional condition. However, the structure of formulas (2.1) does not permit this. We recall that the theory is constructed in a manner which results in a sharp separation between potential and turbulent flow, and this is precisely its merit.

In spite of the lack of clarity noted above, the computations of streams and wakes using the generalized Karman theory have not been without success. In every special case however, various possible assumptions of one sort or another have had to be made (e.g. specifying a point of inflection on the velocity profile and assuming that this point coincides with the maximum of the curve tangential stresses). A remote wake was computed /14/ in this spirit, and it was shown there, in particular, that the turbulent viscosity curve has a form intermediate between the curves produced using the first and second Prandtl theory, which agreed better with experiment.

Moreover, it was found possible, while carrying out the computations for $n = 1$ (corresponding to the Karman theory), to compute the Schlichting constant β from the given value of the Karman constant $\varkappa_1 = 0.16$, thus proving that the physical constants used in the theory of boundary layer turbulence can also be used to solve the free turbulent flows. The most important aspect of this is, that up till now the constants of boundary layer turbulence and free turbulence were regarded as independent. If $n = {}^3/_4$ and $n = {}^2/_3$ are the "working" values of the exponent in (2.1) for computing boundary layer flows, then for free turbulence $n = 1, \varkappa_1 = 0{,}16$ (thus passing to the classical form of the Karman equation), or $n = 0{,}9 \sim 0.95$. should be used. This follows from the fact that the energy dissipation is much slower in free turbulent flows than in boundary layer flows, and the dependence of the turbulent viscosity on the molecular viscosity must be less. However, the fact that it can be taken into account at all (within the framework of the generalized Karman theory) is of some interest.

8. The generalized Karman theory described above belongs, by virtue of its applicability, to the "coarse" versions of the theory of turbulence. It limits itself to considering only steady-state flow, collecting only the most important features and neglecting certain second-order phenomena. Its undoubted shortcoming lies in the fact that solving any particular problem with its help requires a priori discussion of the range of Reynolds numbers within which the solution can be used, even though these ranges are usually fairly wide.

Its merit is its relative simplicity, which produces results easy to interpret for such complex problems as the flow between two rotating cylinders, plane equilibrium boundary layers, flows in curved channels, and Hartman flow. Most of the work on turbulence dealing with specific types of flow (including those mentioned above) is characterized by a large degree of arbitrariness in choosing the formulas for the mixing length and the constants appearing in them. None of this occurs in the proposed method which at all times keeps the same pairs of constants, namely $n = {}^3/_4, \varkappa_n = 0{,}53$, and $n = {}^2/_3, \varkappa_n = 0{,}56$ for flows with relatively low Reynolds numbers. The same values of the empirical constants are retained in the case of curvilinear flows, but are supplemented by another two constants (introduced to account for a new physical factor, e.g. the centrifugal forces which substantially affect turbulent mixing).

Thus the formulas for turbulent viscosity (2.1) embrace a wide class of problems of the theory of turbulence, while maintaining stable empirical constants and a moderate number of them. This must be regarded as a reflection of the fact that the theory in question has a deep physical basis and will, undoubtedly in time be derived from more general relations and ideas. Even now we can see that the basis of the theory is the hypothesis of selfsimilarity of turbulent flows in a form conforming to the fact that the relation between the coefficients of resistance and Reynolds numbers, in the form of Blasius' formula, holds true for many steady-state turbulent flows in ranges of values of the Reynolds numbers of practical interest.

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ON THE CONDITIONS FOR THE ONSET OF MOTION OF TWO COLLINEAR DISLOCATION DISCONTINUITIES*

A.S. BYKOVTSEV

The conditions under which the motion begins of two collinear dislocational Volterra-type discontinuities, initally specified on a single straight line in a homogeneously isotropic elastic medium, is studied. The theory of invariant Γ-integrals /1/ is used to write the criteria defining the beginning and direction of motion of either end of the discontinuity. The limiting stresses are determined and the subsequent behaviour of the whole system is investigated.

Let two generalized dislocational discontinuities of unequal length and constant sudden change in displacement $\mathbf{b}(b_1, b_2, b_3) = \mathrm{const}$ be distributed along a single straight line.We introduce the rectangular Cartesian coordinate system in such a manner that the Ox-axis coincides with the line on which the discontinuities lie, and denote by $-l_1, -l_2, l_3, l_4$ the abscissas of the ends of the discontinuity. The problem is assumed to be plane. We will determine the critical loads which must be applied to the body in order for at least one end of the discontinuity to begin to move. The problem in question is an analog of the problem discussed in /2/ (on the equilibrium of two collinear cracks) for dislocation discontinuities.

Let us denote by u_x, u_y, u_z the components of the displacement vector along the x, y, z axes respectively, and by $\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{xz}, \sigma_{yz}$ the stress tensor components. We also denote the set of internal points of the segments $(-l_1, -l_2)$ and (l_3, l_4) of the Ox-axis by L, and the set of points of the Ox-axis outside these segments by M. The boundary conditions of the problem have the form

$$[\mathbf{u}] = \mathbf{b} \text{ on } L, \ [\mathbf{u}] = 0 \text{ on } M \tag{1}$$

Problem (1) can be written in the form of the sum of the symmetric, skew-symmetric and anti-plane problems, by expanding the vector $\mathbf{b}(b_1, b_2, b_3)$ in three terms $\mathbf{b}_1(b_1, 0, 0), \mathbf{b}_2(0, b_2, 0), \mathbf{b}_3(0, 0, b_3)$. The boundary conditions will have the form (2), (3) and (4) for the skew-symmetric, symmetric and antiplane problems respectively

$$u_x = {}^1/_2 b_1, \ \sigma_{yy} = 0 \text{ on } L; \ u_x = 0, \ \sigma_{yy} = 0 \text{ on } M \tag{2}$$

$$u_y = {}^1/_2 b_2, \ \sigma_{xy} = 0 \text{ on } L; \ u_y = 0, \ \sigma_{xy} = 0 \text{ on } H \tag{3}$$

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